

Better Prefix Authentication

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Abstract

We present new schemes for solving prefix authentication and secure relative timestamping. By casting a new light on antimotone linking schemes, we improve upon the state of the art in prefix authentication, and in timestamping with rounds of bounded length. Our designs can serve as more efficient alternatives to certificate transparency logs.

1 Introduction

Prefix authentication [15] asks to annotate finite sequences with metadata so that one can cryptographically prove when some sequence is a prefix of another. It generalizes *tamper-evident logging* [6] and *secure timestamping* [8].

In recent years, *certificate transparency* [11] and related transparency logging schemes [7] [16] [1] have garnered significant attention. There has been little progress on improving the efficiency of the underlying prefix authentication schemes however.

We provide the first prefix authentication schemes that asymptotically (and practically) outperform the certificate transparency logs, both in the amount of required metadata and in the size of the data that certifies that some string is a prefix of another.

Section 2 summarizes related work, section 3 gives the preliminaries to make sense of what follows. Section 4 introduces our core idea of using slightly modified skip lists, exemplified by a *binary* skip list. Section 5 presents the more efficient *ternary* skip list. We discuss the results in section 6 before concluding in section 7.

Finally, a note on presentation style. We feel like our results could and should have been discovered two decades ago, had the literature on secure timestamping via linking schemes been more accessible to distributed systems practitioners. So we deliberately focus on concrete schemes rather than abstract theory, keep the prose compact and the figures plentiful, and favor intuitive, high-level reasoning over formalisms. Certificate transparency logs are less elegant than linking schemes

(and less efficient as we are about to show), so we hope the ideas stick around this time. Those readers who wish for a more rigorous, formal treatment of prefix authentication via hash-labeled graphs can find it in [15].

2 Related Work

Prefix authentication [15] unifies several previously disjoint strands of research, such as secure logging [20] [6] [19], accountable shared storage [12] [23], certificate transparency [9] [10] [11], or data replication [17] [21]. Our designs fall in the class of *transitive prefix authentication schemes*, more specifically they are *linking schemes*.

Our two linking schemes are almost isomorphic to the *simple antimotone binary linking scheme* [4] and the *optimal antimotone binary linking scheme* [3] respectively. The latter paper introduces the *antimotone graph product* \otimes as an operation that generates all antimotone binary graphs. While we do not explicitly define or rely on this operator, we occasionally mention it to point the interested reader to the parallels to these works.

The simple and optimal antimotone binary linking schemes rely on the same underlying graphs as ours, but the way these graphs are then used to provide prefix authentication is suboptimal (as is their direct transformation into an *anti-monotone ϕ -scheme* [5]).

Threaded authentication trees [5], *hypercore* [17], and *certificate transparency logs* [11] are the most efficient prefix authentication schemes to date. We outperform each of them.

We derive our schemes not from antimotone theory but from slightly modified, deterministic skip lists [18]. Chainiac [16] is a prior approach to prefix authentication based on skip lists, but a significantly less efficient one. Blibech and Gabillon [2] have proposed a skip list timestamping scheme of similar efficiency as ours, but their scheme relies on knowing the maximum length of a timestamping round. As such, it is only applicable to prefix authentication for sequences of bounded length, but not for sequences of arbitrary length.

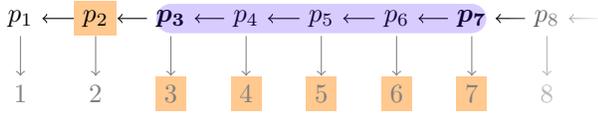


Figure 1: The linear linking scheme. The **out-neighborhood** of the **path from $\text{digest_vertex}(7)$ to $\text{digest_vertex}(3)$** determines the label of $\text{digest_vertex}(7)$: the labels of p_2 and 3 determine the label of p_3 , this together with the label of 4 determines the label of p_4 , and so on.

3 Preliminaries

Rather than laying out the general theory of transitive prefix authentication schemes [15], we present a simple subset that captures the schemes we will present. The results from the underlying theories still apply, but we get to significantly simplify our presentation.

We assume basic understanding of cryptographic hash functions [13], and a basic background in graph theory [22]; we consider *directed* graphs exclusively.

When an object contains a secure hash of another object, the second object must have already existed at the creation time of the first object. This property transitively carries over, and forms the basis of our techniques.

We can represent objects that contain hashes of other objects as labeled DAGs, generalizing the well-known Merkle trees [14]: sinks are labeled with the hashes of some objects of interest (say, the elements of a sequence to authenticate), the other vertices are labeled with the hash of the concatenation of their out-neighbors' labels.

For our purposes, a *linking scheme* is an acyclic graph whose sinks are the natural numbers without zero, in which every sink n has exactly one parent vertex, denoted $\text{digest_vertex}(n)$, and in which there is a path from every $\text{digest_vertex}(n+1)$ to $\text{digest_vertex}(n)$. Figure 1 depicts the simplest such graph, fig. 2 shows the more sophisticated *simple antimonotone binary linking scheme* [4].

Hash-labeled linking schemes can provide prefix authentication. Let t be a sequence of length len_t , then label each sink $n \leq \text{len}_t$ with the hash of the n -th sequence item. For any $\text{len}_s \leq \text{len}_t$, $\text{digest_vertex}(\text{len}_s)$ authenticates the prefix s of length len_s , since all $n \leq \text{len}_s$ are reachable from it and thus influence its hash.

It hence suffices to certify that $\text{digest_vertex}(\text{len}_s)$ is reachable from $\text{digest_vertex}(\text{len}_t)$ in order to prove that s is a prefix of t . This *prefix certificate of s and t* consist of the labels of the closed out-neighborhood of a (shortest) path from $\text{digest_vertex}(\text{len}_t)$ to $\text{digest_vertex}(\text{len}_s)$: these labels suffice to reconstruct the label of $\text{digest_vertex}(\text{len}_t)$, thus proving that there is indeed a path from $\text{digest_vertex}(\text{len}_t)$ to $\text{digest_vertex}(\text{len}_s)$ (compare fig. 1).

In order to evaluate the efficiency of a linking scheme,

we ask for a mapping $\text{certificate_pool} : \mathbb{N}^{\geq 1} \rightarrow \mathcal{P}(V)$ that maps every number to a set of vertices such that for all $\text{len}_s < \text{len}_t$ the shortest path from $\text{digest_vertex}(\text{len}_t)$ to $\text{digest_vertex}(\text{len}_s)$ is contained in the intersection $\text{certificate_pool}(\text{len}_s) \cup \text{certificate_pool}(\text{len}_t)$. The out-neighborhood of $\text{certificate_pool}(n)$ is called the *positional certificate* of n .

A particularly interesting class of linking schemes, due to their similarity to our schemes, are the *antimonotone binary linking schemes* [4] [3]. In these schemes, the parent of sink n — denoted p_n — has an edge to p_{n-1} , and another edge to $p_{f(n)}$, where f is some function that satisfies the *antimonotonicity property* $n < m \implies f(n) \geq f(m)$. Figure 2 depicts the *simple antimonotone binary linking scheme* [4], whose graph G_{ls2} is given by the following function f_2 :

$$f_2(n) := \begin{cases} n - (2^{k-1} + 1) & \text{if } n = 2^k - 1, k \in \mathbb{N} \\ n - 2^{g(n)} & \text{otherwise} \end{cases}$$

$$g(n) := \begin{cases} k & \text{if } n = 2^k - 1, k \in \mathbb{N} \\ g(n - (2^{k-1} - 1)) & \text{if } 2^{k-1} - 1 < n < 2^k - 1, k \in \mathbb{N}. \end{cases}$$

4 Skip List Linking Schemes Done Right

Let $n, k \in \mathbb{N}_0$, then $\text{maxpow}_k(n)$ denotes the largest natural number p such that k^p divides n .

We now present the *SLLS₂ (binary skip list linking scheme)*. Its underlying graph is a skip list with a twist, links that would normally stay in the same layer of the skip list point to the topmost available layer instead (fig. 3):

$$V_{slls2} := \{(n, k) : n \in \mathbb{N}, k \in \mathbb{N}_0 \text{ and } 2^k \mid n\} \cup \mathbb{N},$$

$$E_{slls2} := \left\{ ((n, k+1), (n, k)) \right\} \cup \left\{ ((n, 0), n) \right\} \\ \cup \left\{ ((n+2^k, k), (n, \text{maxpow}_2(n))) \right\},$$

$$G_{slls2} := (V_{slls2}, E_{slls2}).$$

Perhaps surprisingly, the *SLLS₂* is almost isomorphic to the simple antimonotone binary scheme [4], compare fig. 3 and fig. 2. More precisely, $G_{slls2} \setminus \mathbb{N}$ is isomorphic to $G_{ls2} \setminus \mathbb{N}$ (and both are isomorphic to the limit of the recursive antimonotone product $G_{simple}^{i+1} := G_{simple}^i \otimes G_{simple}^i \otimes G_1$) [3].

As a single look at fig. 3 and fig. 2 leaves little doubt about the isomorphism, we give no further proof beyond stating the bijective function between the vertices. A vertex (n, k) of the *SLLS₂* maps to the vertex m of G_{ls2} , where m is the number of vertices on the longest path from (n, k) to $(1, 0)$. Conversely, a vertex m of G_{ls2} maps to the vertex (n, k) of the *SLLS₂*, where n is the number of edges in the longest path from m to 1 of the form $(x, f_2(x))$, and k is the number of consecutive edges at

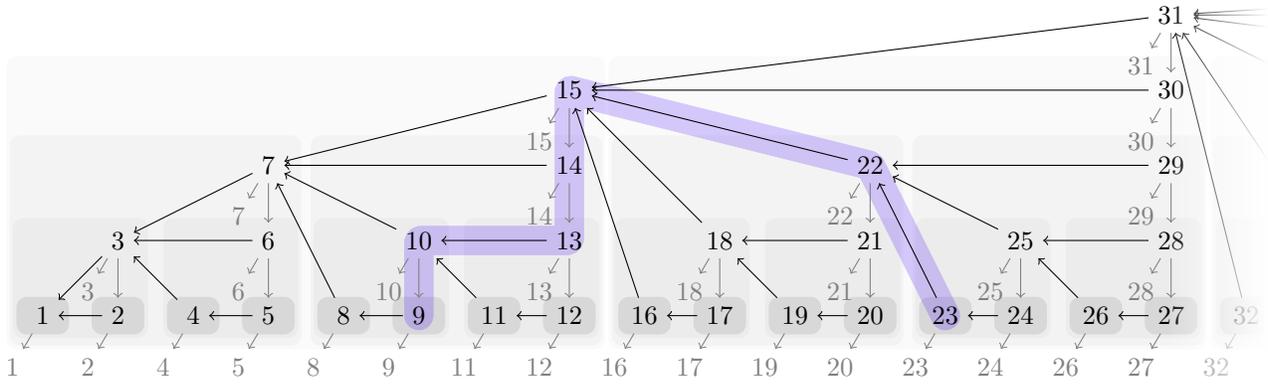


Figure 2: The simple antimonotone binary linking scheme G_{ls2} , highlighting the shortest path from 23 to 9.

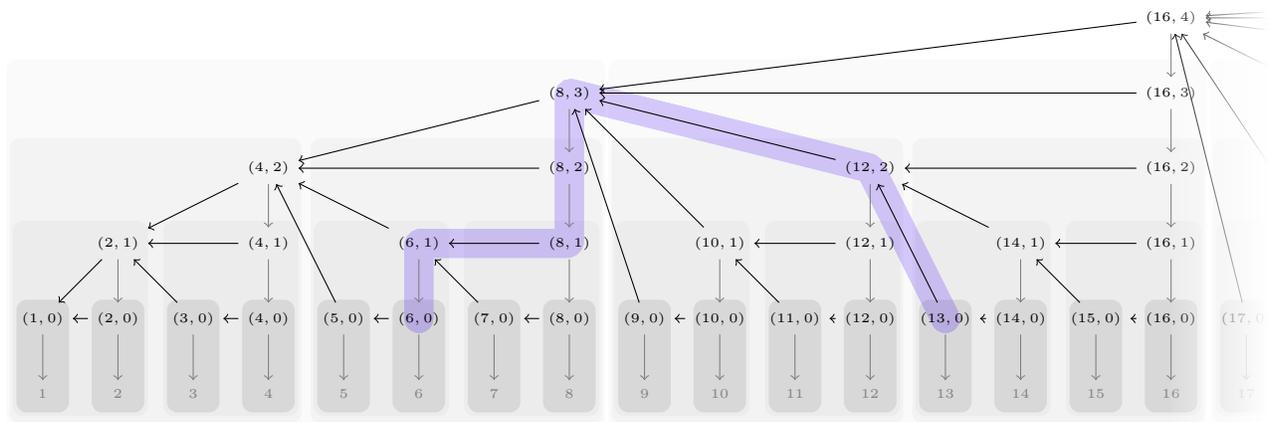


Figure 3: The $SLLS_2$, highlighting the shortest path from (13, 0) to (6, 0).

the start of the longest path from m to 1 of the form $(x, x - 1)$. In less formal terms, (n, k) are the x and y coordinates of m when drawing G_{IS_2} in the style of fig. 2.

The certificate pools for $SLLS_2$ correspond directly to those of G_{IS_2} under the isomorphism. For any number n , we say it belongs to *generation* $\lceil \log_2(n) \rceil$. We say $(t, \log_2(t))$ is the *vertebra* of generation t , and the set of vertebrae of all generations up to and including t is the *spine* of generation t . The certificate pool of n is the union of the shortest paths from the vertebra of t to $(n, 0)$, from $(n, 0)$ to the vertebra of $t - 1$, and from that vertebra to $(1, 0)$. In the isomorphic antimonotone setting, Buldas et al. [4] have proven that the intersection of two such certificate pool contains the shortest path between the numbers in question, so the construction does indeed yield a valid certificate pool. Figure 4 and fig. 5 convey the underlying intuition.

The size of the out-neighborhood of the certificate pool of some n is different to that in G_{IS_2} however; we claim it is $2 \cdot \lceil \log_2(n) \rceil$ for all $n > 4$. Let $n \in \mathbb{N}, n > 4$, and let t be the generation of n . The shortest path from the vertebra of $t - 1$ to $(1, 0)$ consists of the vertices $(2^{t-1}, t - 1), (2^{t-2}, t - 2), \dots, (2^0, 0)$, for a total of t vertices. Every such vertex has exactly one out-neighbor outside the path: 1 for $(1, 0)$, and $(x, k - 1)$ for any other (x, k) . This brings the out-neighborhood of the certificate pool to a size of at least $\lceil \log_2(n) \rceil$. It remains to show that the out-neighborhood of the shortest path from the vertebra of t to the vertebra of $t - 1$ via $(n, 0)$ has the same size.

We have two different kinds of outgoing edges to consider: those from some (x, k) to $(x, k - 1)$, and those whose first component decreases by some amount. From the antimonotone perspective, the latter are those of the form $(n, f_2(n))$, and the remaining edges correspond to those of form (x, k) to $(x, k - 1)$. We will call the edges of form $(n, f_2(n))$ *jump edges*, and the other edges *predecessor edges*. In our figures, the predecessor edges are deemphasized.

The antimonotonicity of G_{IS_2} implies that every jump edge of a vertex in the shortest path from some m to 1 leads into that path again. Assume toward a contradiction there is a jump edge (z, x) such that z is part of the path but x is not. Then the path must contain some other jump edge (y, w) with $w < x < y$, contradicting antimonotonicity. By the isomorphism, it immediately follows that all jump edges from vertices of the certificate pool of n lead into the certificate pool again, and thus do not enlarge its out-neighborhood.

To analyze the predecessor edges, we define the *core* of generation t as the subgraph of G_{SLLS_2} induced by the longest path from $(2^t, t - 2)$ to $(2^{t-3}, t - 3)$, the *start* of (the core of) t as $(2^t, t - 2)$, and the *middle* of (the core of) t as $(2^{t-1}, t - 2)$ — see fig. 6 for some examples. Figure 7 visualizes the recursive structure of cores: the core of generation t consists of two copies of the core of generation $t - 1$.

This recursive structure allows us to compute the number of predecessor edges on the shortest path from vertebra to vertebra via some $(n, 0)$. We exclude the out-neighbors of the

final vertex of the path, as those are already accounted for in our calculation for the size of the path from the vertebra of $t - 1$ to $(1, 0)$.

Let $n \in \mathbb{N}, n > 4$ be of generation $t > 2$, then the vertebra of t is $(2^t, t)$, the **start of the core** is $(2^t, t - 2)$, the **middle of the core** is $(2^{t-1}, t - 2)$, and the **previous vertebra** is $(2^{t-1}, t - 1)$. The skip edges of $(2^t, t)$ and $(2^t, t - 1)$ lead out of the current generation, so the path to $(n, 0)$ has to begin with $((2^t, t), (2^t, t - 1), (2^t, t - 2))$, and hence, the predecessor edges of both $(2^t, t)$ and $(2^t, t - 1)$ stay within the path.

The behavior of the predecessor edge of $(2^t, t - 2)$ depends on whether n falls into the first or second half of the core (fig. 8). If $n > 2^t - 2^{t-1}$, the path continues to $(2^t, t - 3)$, so the predecessor edge of $(2^t, t - 2)$ leads into the path. In this case, however, the path to $(2^{t-1}, t - 1)$ ends with $((2^t - 2^{t-1}, t - 2), (2^{t-1}, t - 1))$, so the predecessor edge of $(2^t - 2^{t-1}, t - 2)$ leads outside the path. If $n \leq 2^t - 2^{t-1}$, the path continuous from $(2^t, t - 2)$ to $(2^t - 2^{t-1}, t - 2)$, so the predecessor edge of $(2^t, t - 2)$ leads outside the path.

In both cases, the size of the out-neighborhood is one plus the out-neighborhood of the corresponding path in the core of generation $t - 1$. Together with the base case of an out-neighborhood of size two for $t = 2$, induction yields an overall size of $t = \lceil \log_2(n) \rceil$. Thus, the out-neighborhood of the certificate pool of any $n > 4$ is indeed $2 \cdot \lceil \log_2(n) \rceil$.

4.1 Timestamping Rounds

While not our primary focus, we briefly sketch how the $SLLS_2$ meets the optimal positional certificate size in the setting of timestamping with rounds of bounded length. We let N denote the round length and T denote the generation of N ; to simplify the presentation we assume that N is a power of two.

Let $n \leq N$ be of generation t . Let s be the vertex just before the vertebra of $t - 1$ on the shortest path from $(n, 0)$ to the vertebra of $t - 1$. Then the certificate pool of n for rounds of length N is the shortest path from $(N, \log_2(N))$ to s via $(n, 0)$. Figure 9 and fig. 10 exemplify how the union of two certificate pools contains the shortest path between their two numbers, whether the numbers are from the same generation or not.

In the previous section, we have shown that the size of the out-neighborhood of the shortest path from the vertebra of t to the vertebra of $t - 1$ via $(n, 0)$ — excluding the out-neighbors of the vertebra of $t - 1$ — is $\lceil \log_2(n) \rceil$. In this new setting, the path stops at s already, so the size of the out-neighborhood increases by one, as the vertebra of $t - 1$ itself becomes part of the out-neighborhood.

If $t = T$, this path is the certificate pool of n , and it thus has size $\lceil t \rceil + 1 = \lceil \log_2(N) \rceil + 1$. If $t < T$, the certificate pool of n consists of the shortest path from $(N, \log_2(N))$ to the vertebra of t , followed by the shortest path from the vertebra of t to the vertebra of $t - 1$ via $(n, 0)$. The latter has size $t + 1$, and the prior is a path of length $T - t$, and each of

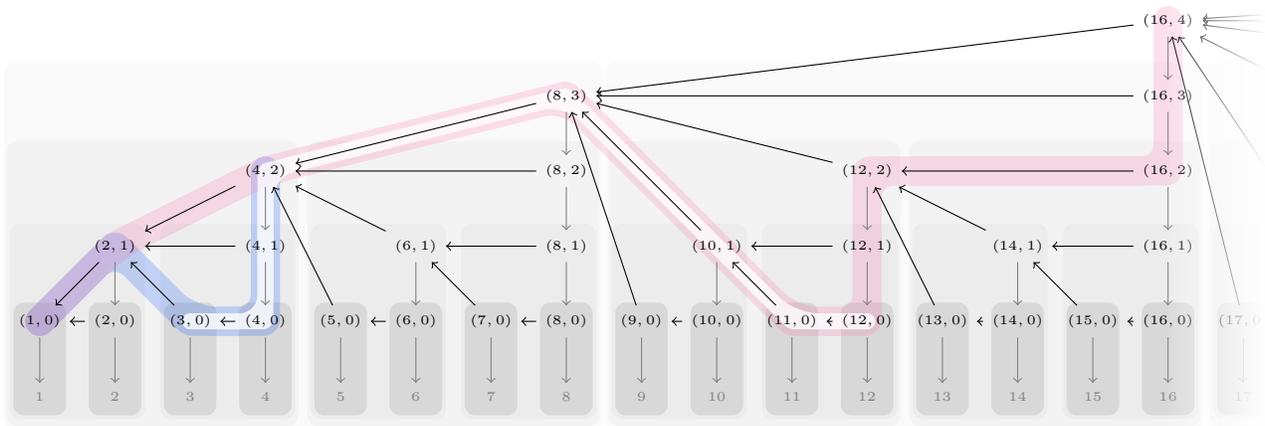


Figure 4: The certificate pools of 3 and 12, two numbers from different generations. The vertebra of the generation of the lesser number lies on the spine of the generation of the greater number.

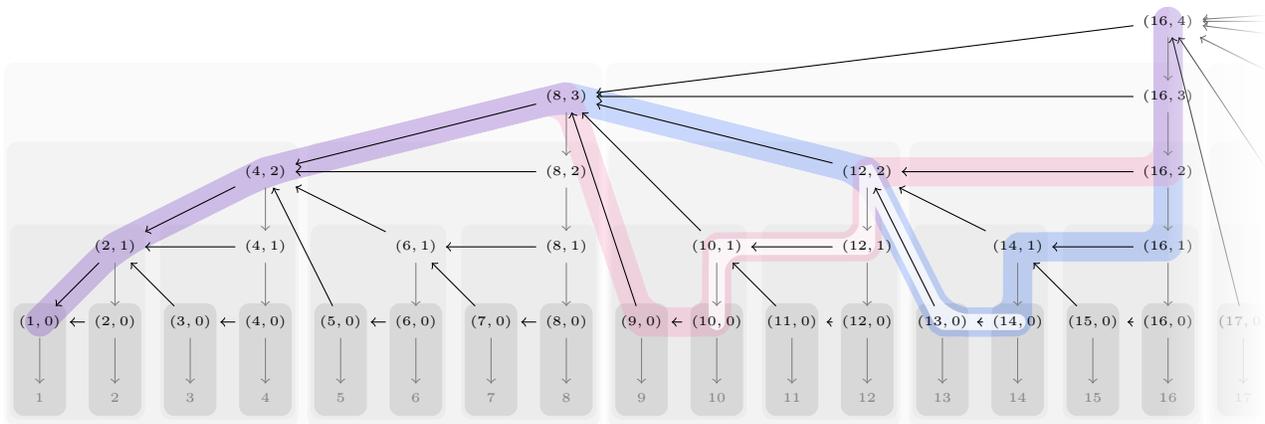


Figure 5: The certificate pools of 10 and 14, two numbers from the same generation. Their paths necessarily cross within the generation.

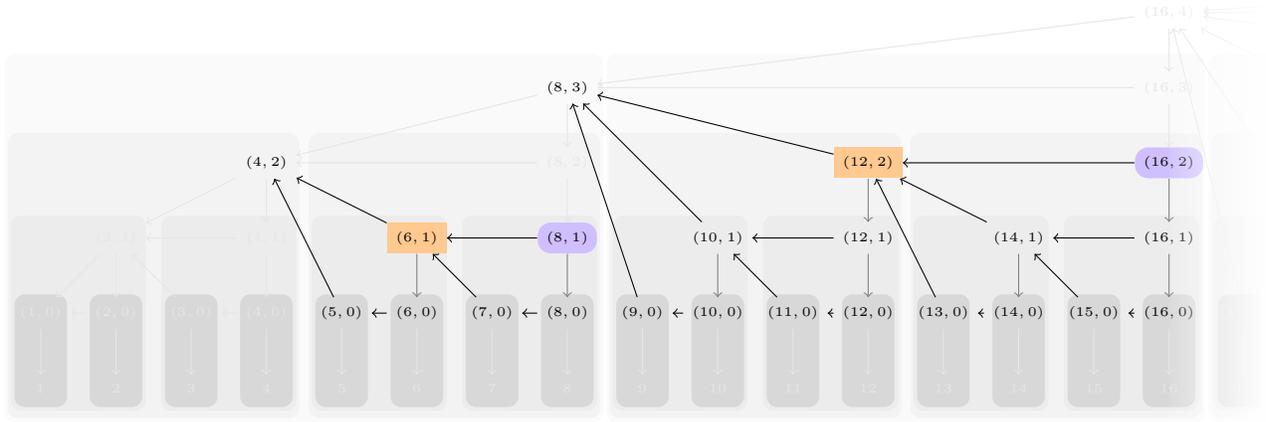


Figure 6: The cores of generations three and four for $SLLS_2$, highlighting their starts and middles.

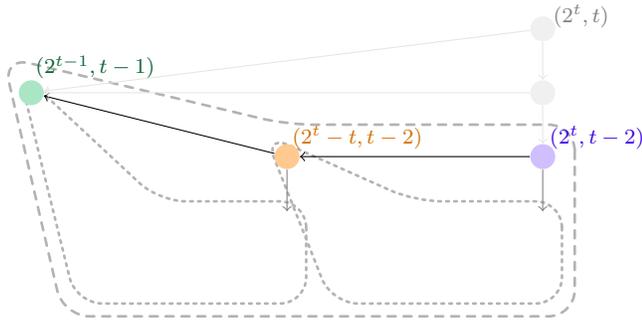


Figure 7: The recursive structure of cores for $SLLS_2$: the core of generation t (dashed) consists of two copies of the core of generation $t-1$ (dotted).

its vertices increases the size of the out-neighborhood by one, except for the final vertex, whose two out-neighbors are already counted or included in the path from the vertebra of t to the vertebra of $t-1$ via $(n, 0)$. As such, the total size is $t+1 + (T-t) = T+1 = \lceil \log_2(N) \rceil + 1$ again.

When we add an edge from each $(n, 0)$ to the final vertex of the previous timestamping around for inter-round timestamping, we obtain the final positional certificate size: $(\lceil \log_2(N) \rceil + 2) \cdot k$, where k is the size of an individual hash. This size is provably optimal [5].

5 Ternary Skip List Scheme

Choosing a base other than 2 in the definition of a skip-list linking scheme results again in a linking scheme. Of particular interest is base 3, as this turns out to yield a more efficient construction in the round-less setting, the *ternary skip list*

linking scheme ($SLLS_3$):

$$\begin{aligned}
 V_{slls3} &:= \{(n, k) : n \in \mathbb{N}, k \in \mathbb{N}_0 \text{ and } 3^k \mid n\} \cup \mathbb{N}, \\
 E_{slls3} &:= \left\{ ((n, k+1), (n, k)) \right\} \\
 &\quad \cup \left\{ ((n+3^k, k), (n, \text{maxpow}_3(n))) \right\} \\
 &\quad \cup \left\{ ((n+2 \cdot 3^k, k), (n+3^k, \text{maxpow}_3(n))) \right\}, \\
 G_{slls3} &:= (V_{slls3}, E_{slls3}).
 \end{aligned}$$

Figure 11 depicts the graph.

The remaining discussion is completely analogous to that of the $SLLS_2$.

The $SLLS_3$ (without the sinks) is isomorphic to the *optimal antimotone linking scheme* [3] (without the sinks), and to the limit of the recursive antimotone graph product $G_{opt}^{i+1} := G_{opt}^i \otimes G_{opt}^i \otimes G_{opt}^i \otimes G_1$.

We define the *generation* of n as $\lceil \log_3(n) \rceil$. We say $(t, \log_3(t))$ is the *vertebra* of generation t , and the set of vertebra of all generations up to and including t is the *spine* of generation t . The certificate pool of n is the union of the shortest paths from the vertebra of t to $(n, 0)$, from $(n, 0)$ to the vertebra of $t-1$, and from that vertebra to $(1, 0)$.

This yields positional certificates of size $3 \cdot \lceil \log_3(n) \rceil$; the proof is analogous to that for $SLLS_2$. Let n be of generation t , then the out-neighborhood of the path from the vertebra of $t-1$ to $(1, 0)$ has size t .

For the remaining path, we again consider *cores*, this time beginning at $(3^t, t-2)$ and being split into *three* parts at the vertices $(3^t - 3^{t-1}, t-2)$ and $(3^t - 2 \cdot 3^{t-1}, t-2)$. Each of these parts is isomorphic to the core of generation $t-1$, allowing us to apply induction on t (see fig. 12). The shortest path from the vertebra of t to the vertebra of $t-1$ via $(n, 0)$

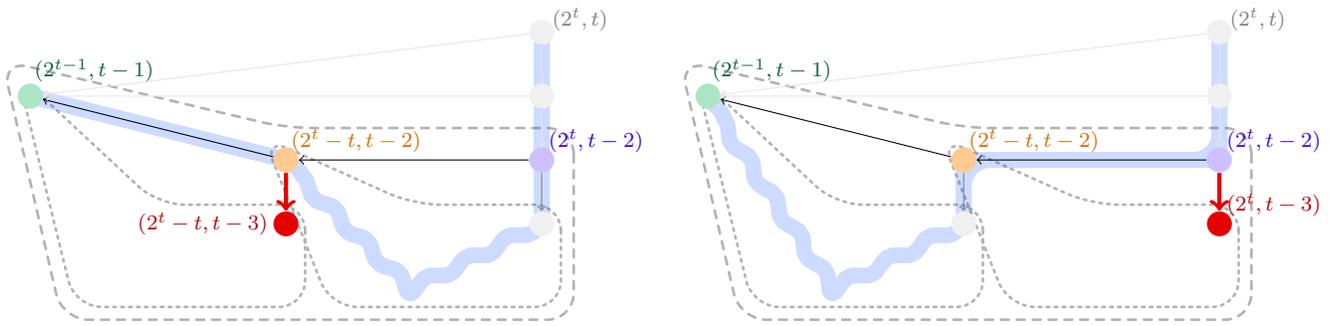


Figure 8: The two possible path shapes in the inductive step. Whether n is in the first or second half of the core, there is exactly **one new out-neighbor** of the path compared to the path through the core of the previous generation.

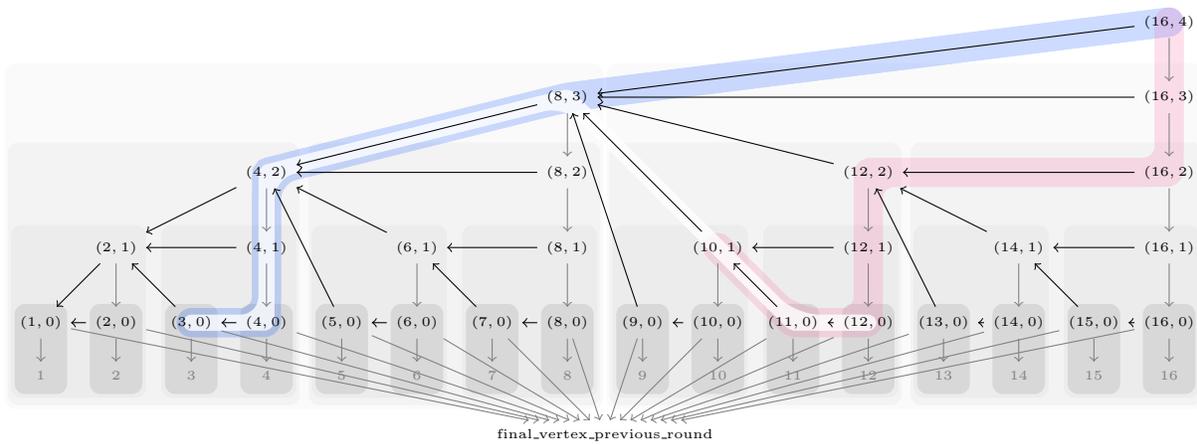


Figure 9: The certificate pools of 3 and 12, two numbers from different generations, in a timestamping round of size $N = 16$.

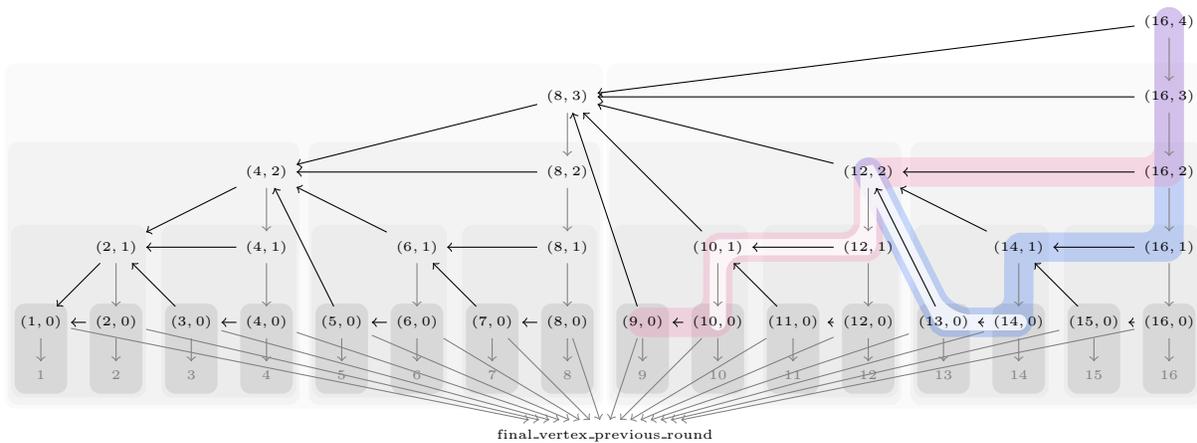


Figure 10: The certificate pools of 10 and 14, two numbers from the same generation, in a timestamping round of size $N = 16$.

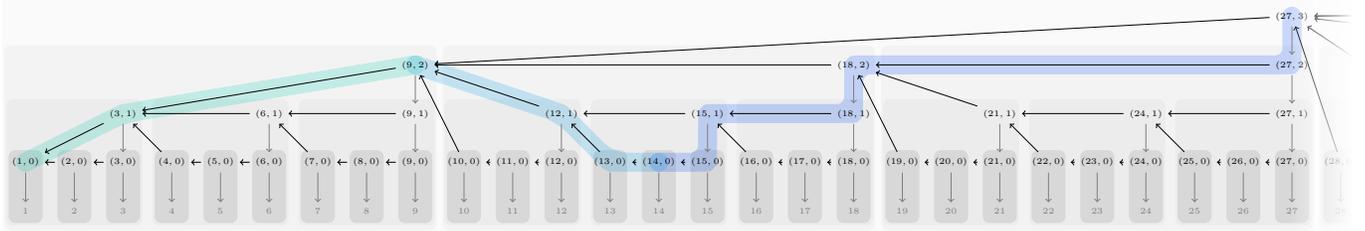


Figure 11: The $SLLS_3$, highlighting the certificate pool of 14.

descends into one of the three parts of the core, the predecessor edges of the start vertices of the other two parts lead outside the path. Hence, the size of the out-neighborhood of this path is $2 \cdot t$, for an overall positional certificate size of $3 \cdot t = 3 \cdot \lceil \log_3(n) \rceil$.

This makes $SLLS_3$ positional certificates about 95 percent the size of $SLLS_2$ positional certificates, as $\frac{3 \cdot \log_3(n)}{2 \cdot \log_2(n)} = \frac{\log_e 8}{\log_e 9} \approx 0.9464$. Table 1 list the positional certificate sizes for all n up to 2^{33} , to aid with the decision of whether to go with the simpler $SLLS_2$ or the asymptotically more efficient $SLLS_3$. For all $n > 2^{16}$, $3 \cdot \lceil \log_3(n) \rceil$ is strictly less than $2 \cdot \lceil \log_2(n) \rceil$.

We can naturally generalize base-3 skip list construction to arbitrary integer bases; the corresponding antimonotone schemes have been studied by Buldas and Laud [3]. Definitions and proofs for positional certificates generalize nicely, they have size $b \cdot \lceil \log_b(n) \rceil$ for the base- b skiplist scheme. $3 \cdot \lceil \log_3(n) \rceil$ is the slowest-growing function of this family, making the most efficient scheme $SLLS_3$ and the simplest scheme $SLLS_2$ the only two members of this family of practical interest.

6 Discussion

The $SLLS_2$ achieves the same positional certificate size as threaded authentication trees [5], hypercore [17], and certificate transparency logs [9], the best previously known schemes in that respect. Unlike these schemes, it does so with a underlying graph of linear size: every vertex has out-degree at most 2, and the number of vertices of the subgraph for a sequence of length n is at most $3n$. Furthermore, the digest vertex of every sequence item is the direct parent of the sequence item, whereas the distance between them can be logarithmic in hypercore and certificate transparency logs.

As such, the $SLLS_2$ strictly outperforms all existing schemes that achieve positional certificates of size $2 \cdot \lceil \log_2(n) \rceil$. The $SLLS_3$ then further improves upon the positional certificate size, it is the first scheme to beat the positional certificate size of $2 \cdot \lceil \log_2(n) \rceil$. It is worth noting that simply adapting hypercore or transparency logs to a ternary rather than binary structure does *not* result in positional certificates of size $3 \cdot \lceil \log_3(n) \rceil$ (see fig. 13 and fig. 14). The skip-list-with-a-twist is qualitatively different from tree struc-

	\leq	2	3		\leq	2	3
1	1	1		131072	34	33	
2	2	3		177147	36	33	
3	4	3		262144	36	36	
4	4	6		524288	38	36	
8	6	6		531441	40	36	
9	8	6		1048576	40	39	
16	8	9		1594323	42	39	
27	10	9		2097152	42	42	
32	10	12		4194304	44	42	
64	12	12		4782969	46	42	
81	14	12		8388608	46	45	
128	14	15		14348907	48	45	
243	16	15		16777216	48	48	
256	16	18		33554432	50	48	
512	18	18		43046721	52	48	
729	20	18		67108864	52	51	
1024	20	21		129140163	54	51	
2048	22	21		134217728	54	54	
2187	24	21		268435456	56	54	
4096	24	24		387420489	58	54	
6561	26	24		536870912	58	57	
8192	26	27		1073741824	60	57	
16384	28	27		1162261467	62	57	
19683	30	27		2147483648	62	60	
32768	30	30		3486784401	64	60	
59049	32	30		4294967296	64	63	
65536	32	33		8589934592	66	63	

Table 1: The number of vertices in the out-neighborhood of the certificate pools for all $n \leq 2^{33}$ of $SLLS_2$ and $SLLS_3$. For example, for $64 < n \leq 81$, the out-neighborhood has size 14 for $SLLS_2$ and size 12 for $SLLS_3$. The highlighted rows are those where $SLLS_2$ is more efficient than $SLLS_3$.

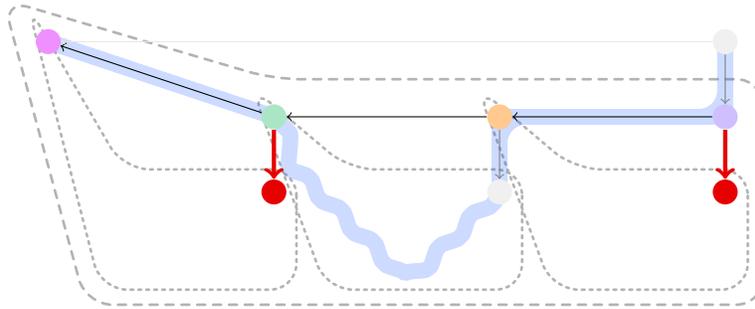


Figure 12: The recursive structure of the cores of $SLLS_3$, each consisting of three copies of the core of the previous generation. No matter in which of the sub-cores a vertex $(n, 0)$ lies, the start vertices of the other two sub-cores increase the size of the out-neighborhood of the certificate pool by two compared to the previous generation.

tures in that regard.

For timestamping with rounds of bounded length, $SLLS_2$ is the first scheme to achieve minimal structural certificates with a graph of linear size.

Table 2 takes the (competitive) schemes surveyed in [15] and compares them to our new schemes. The picture is quite clear: if any new sequence item must add no more than $O(1)$ of metadata, use the optimal antimonotone linking scheme. In every other case, use the $SLLS_2$ (if using rounds of bounded length) or the $SLLS_3$.

7 Conclusion

We have provided prefix authentication schemes that outperform the previous state of the art. We hope that future endeavors around certificate transparency in particular will adopt our schemes.

We did not consider questions of optimality, leaving open whether schemes with smaller positional certificates exist.

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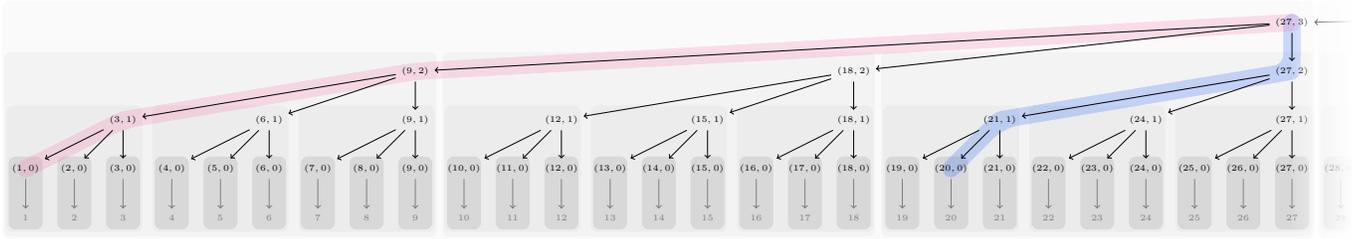


Figure 13: Basing hypercore-like or transparency-log-like schemes off ternary trees is inefficient: the paths from a root to the two leaves each have 2 out-neighbors per vertex, for an overall out-neighborhood size of $4 \cdot \lceil \log_3(n) \rceil - 1$ rather than the desired $3 \cdot \lceil \log_3(n) \rceil$.

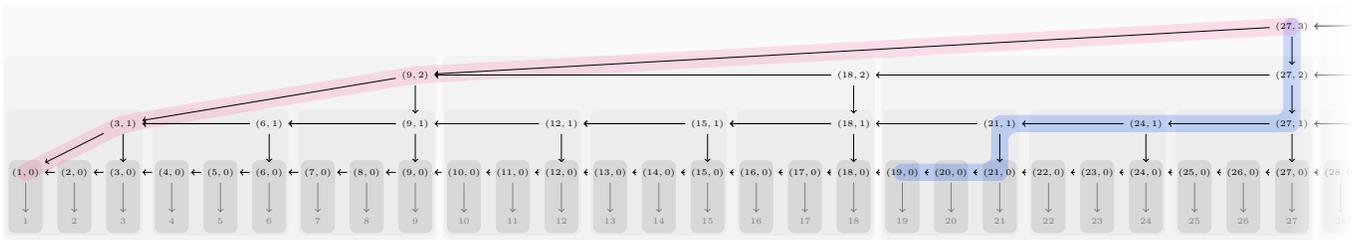


Figure 14: A conventional ternary skip list is inefficient as well: while the path to (1, 0) only contributes one out-neighbor per vertex, the other path contributes three out-neighbors per level of the skip list.

	Linear	Simple Antimonotone	Optimal Antimonotone	Threaded Authentication
Positional Certificate	$n \cdot k$	$(5 \cdot \lceil \log_2(n) \rceil - 3) \cdot k$	$(7 \cdot \lceil \log_3(2n) \rceil - 4) \cdot k$	$2 \cdot \lceil \log_2(n) \rceil \cdot k$
Certificate Validation	$O(\text{certsize})$	$O(\text{certsize})$	$O(\text{certsize})$	$O(\text{certsize} \cdot \log(\text{certsize}))$
Edges Amortized	$O(n)$	$O(n)$	$O(n)$	$O(n \cdot \log(n))$
Edges Worst Case	$O(1)$	$O(1)$	$O(1)$	$O(\log(n))$
Vertices Amortized	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Vertices Worst Case	$O(1)$	$O(1)$	$O(1)$	$O(\log(n))$
Identifier Amortized	$O(1)$	$O(1)$	$O(1)$	$O(1)$
Identifier Worst Case	$O(1)$	$O(1)$	$O(1)$	$O(1)$
Digest Pool	1	$\lceil \log_2(n) \rceil$	$\lceil \log_3(2n) \rceil$	$\lceil \log_2(n) \rceil$
	Hypercore	Transparency Log	$SLLS_2$	$SLLS_3$
Positional Certificate	$2 \cdot \lceil \log_2(n) \rceil \cdot k$	$2 \cdot \lceil \log_2(n) \rceil \cdot k$	$2 \cdot \lceil \log_2(n) \rceil \cdot k$	$3 \cdot \lceil \log_3(n) \rceil \cdot k$
Certificate Validation	$O(\text{certsize})$	$O(\text{certsize})$	$O(\text{certsize})$	$O(\text{certsize})$
Edges Amortized	$O(n \cdot \log(n))$	$O(n \cdot \log(n))$	$O(n)$	$O(n)$
Edges Worst Case	$O(\log(n))$	$O(\log(n))$	$O(1)$	$O(1)$
Vertices Amortized	$O(n)$	$O(n \cdot \log(n))$	$O(n)$	$O(n)$
Vertices Worst Case	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$
Identifier Amortized	$O(1)$	$O(1)$	$O(1)$	$O(1)$
Identifier Worst Case	$O(\log(n))$	$O(\log(n))$	$O(1)$	$O(1)$
Digest Pool	$\lceil \log_2(n) \rceil$	$\lceil \log_2(n) \rceil$	$\lceil \log_3(2n) \rceil$	$\lceil \log_2(n) \rceil$

Table 2: Summary of prior competitive schemes and our new schemes, according to the complexity criteria layed out in [15].

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